International Journal of Ethics in Engineering & Management Education
Website: www.ijeee.in (ISSN: 2348-4748, Volume 2, Issue 1, January 2015)

Comparison and Correlation Coefficient between CRS and VRS models of OC Mines

Dr.G.Thirupati Reddy
Principal & Professor,
Dept of Mechanical Engineering,
Sree Visvesvaraya Institute of Tech. & Science,
Mahabubnagar, Telengana State, India

Abstract: Data Envelopment Analysis (DEA) is a relatively new "data oriented" approach for evaluating the performance of a set of peer entities called Decision Making Units (DMUs) which convert multiple inputs into multiple outputs. DEA is a multi-factor productivity analysis model for measuring the relative efficiency of a homogenous set of coal mines (DMU's). Coal accounts for 50% of total commercial energy supplied in India. The rising demand for coal and the inability of the domestic coal production to meet the demand is a challenging task to improve the productivity and reducing cost. Productivity improvement and cost control have become key objectives of Singareni Collieries Company Limited (SCCL) coal mines in recent years. This paper uses Data Envelopment Analysis (DEA) technique which can aggregate the input and output components in such situations for obtaining an overall performance measure. For every inefficient coal mine, DEA identifies a set of corresponding efficient coal mines that can be utilized as benchmarks for improvement of performance and productivity. DEA developed based on two scale of assumptions viz., **Constant Return to Scale (CRS**) model and **Variable Return to Scale (VRS)** model. Comparison i**s made between CRS and VRS models.**

Key words: **DEA, CRS Method, VRS Method, Correlation coifficient**

INTRODUCTION

With DEA, the efficient frontier is the benchmark against which the relative performance of firms is measured. Given a certain sample of firms, all companies should be able to operate at an optimal efficiency level, which is determined by the efficient companies in the sample. These efficient companies are usually referred as the "peer firms" and determine the efficiency frontier. The companies that form the efficient frontier use minimum quantity of inputs to produce the same quantity of outputs.

PRODUCTIVITY IN COAL MINES (OMS)

Historically and traditionally productivity in coal mines all over the world is being measured in terms of output per man shift (OMS). Thus, productivity (OMS) in underground mines is
= Tonnage of coal produced / Manpower x Shifts

For opencast mines also productivity is measured in terms of OMS, but additional efforts put to excavate the overburden is also taken into account, The OMS of opencast mines is determined as (Seam and Verma (1992))

P+1.4Q/M (1+1.4R)

Where P = Production of coal in tones

Q = Overburden removed in cubic meter

R = Stripping ratio in cubic meter of over burden per tonne of coal and

1.4 is assumed average specific gravity of coal.

Process Benchmarking can be a powerful tool in the productivity improvement process, since the framework fits nicely into an operational approach to improving performance. Data Envelopment Analysis (DEA) is a relative efficiency measurement approach is receiving increasing importance for performance evaluation and benchmarking. It uses Operations Research Techniques to calculate the relative efficiency.

METHODOLOGY

CRS DEA Model

Input-oriented	
Envelopment model	Multiplier model
$\min \theta - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)$	$\max z = \sum_{r=1}^{s} \mu_r y_{ro}$
subject to	subject to
$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta x_{io} \quad i = 1,2,\ldots,m;$	$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0$
$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro} \quad r = 1,2,\ldots,s;$	$\sum_{i=1}^{m} v_i x_{io} = 1$
$\lambda_j \ge 0 \quad j = 1,2,\ldots,n.$	$\mu_r, v_i \ge \varepsilon > 0$

International Journal of Ethics in Engineering & Management Education
Website: www.ijeee.in (ISSN: 2348-4748, Volume 2, Issue 1, January 2015)

Output-oriented	
Envelopment model	Multiplier model
$\max \phi + \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)$	$\min q = \sum_{i=1}^{m} v_i x_{io}$
subject to	subject to
$\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{io} \quad i = 1,2,\ldots,m;$	$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} \geq 0$
$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = \phi y_{ro} \; r = 1,2,\ldots,s;$	$\sum_{r=1}^{s} \mu_r y_{ro} = 1$
$\lambda_j \geq 0 \qquad j = 1,2,\ldots,n.$	$\mu_r, v_i \geq \varepsilon > 0$

VRS (Variable to Returns Scale) or BCC Model

If the constraint is is adjoined, they are known as BCC (Banker, Cooper, 1984) models. This added constraint introduces an additional variable, into the (dual) multiplier problems. This extra variable makes it possible to effect returns-to-scale evaluations (increasing, constant and decreasing). So the BCC model is also referred to as the VRS (Variable Returns to scale) model and distinguished form the CCR model which is referred to as the CRS (Constant Returns to Scale) model.

The CRS model is designed with the assumption of constant returns to scale. This means that there is no assumption that any positive or negative economies of scale exist. It is assumed is that a small unit should be able to operate as efficiently as a large one – that is, constant returns to scale. In order to address this, Banker, Charnes, and Cooper developed the BCC model. It is also referred as VRS model. The VRS model is closely related to the standard CRS model as is evident in the dual of the BCC model:

$$\min(\theta,\lambda) = \theta$$

$$\text{subject to} \quad \begin{aligned} &\theta x_0 - X\lambda = s^- \\ &Y\lambda = y_0 + s^+ \\ &e\lambda = 1 \\ &\lambda \geq 0, s^+ \geq 0, s^- \geq 0 \end{aligned}$$

The difference compared to the CRS model is the introduction of the convexity condition . This additional constraint gives the frontiers piecewise linear and concave characteristics.

DATA COLLECTION AND ANALYSIS

Table1: Normalized Data for Open-Cast mines

Normalized data of OC mines					
Mines(DMU)	Wage Cost	Store Cost	OBR Cost	Other Cost	Production
OCM1	1.4159	1.3481	1.6260	1.5881	1.4980
OCM2	0.4178	0.2750	1.1271	0.6606	1.0283
OCM3	0.8347	0.3747	0.2395	0.2439	0.4547
OCM4	0.2877	0.0429	0.0886	1.4318	0.9398
OCM5	2.2116	2.7843	1.0544	1.9245	1.6182
OCM6	0.1794	0.3421	0.5946	0.3132	0.6900
OCM7	0.0900	0.0640	0.1193	0.0033	0.1348
OCM8	0.8788	0.6435	2.3050	0.6806	1.2584
OCM9	0.4472	0.3099	1.5266	0.3449	0.7523
OCM10	0.3140	0.1812	0.5095	0.1531	0.4167
OCM11	0.2761	0.0975	0.4884	0.2727	0.4347
OCM12	0.8668	0.4730	1.9179	0.5059	1.3427
OCM13	2.5188	3.8545	1.5713	2.2644	2.1494
OCM14	1.7423	1.7183	0.7791	0.7015	0.8720
OCM15	2.5188	2.4909	1.0527	3.9112	1.4102

Comparison between Input-oriented CRS and Input-oriented VRS Models

The table 2 shows the comparison between these two models based on analysis carried using TORA & DEAP software.

International Journal of Ethics in Engineering & Management Education
Website: www.ijeee.in (ISSN: 2348-4748, Volume 2, Issue 1, January 2015)

Table 2: Comparison between Input-oriented CRS and Input-oriented VRS models of OC Mines

OPEN CAST MINES								
	Efficiency		Peer Groups		Peer Count		Ranking	
DMU	Input-Oriented CRS	Input-Oriented VRS	Input-Oriented CRS	Input-Oriented VRS	Input-Oriented CRS	Input-Oriented VRS	Input-Oriented CRS	Input-Oriented VRS
OCM1	54.96%	96.50%	4	4,12,13	0	0	12	6
OCM2	100.00%	100%	2	2	5	4	3	3
OCM3	100.00%	100%	3	3	5	1	3	5
OCM4	100.00%	100%	4	4	6	6	2	1
OCM5	67.73%	94.20%	3, 4, 7	4,6,13	0	0	10	7
OCM6	100.00%	100%	6	6	3	5	4	2
OCM7	100.00%	100%	7	7	9	3	1	4
OCM8	71.25%	84.90%	2, 6, 7	2,4,12	0	0	8	11
OCM9	85.51%	90.20%	2, 6, 7	2,6,7,12	0	0	6	9
OCM10	83.19%	93.30%	2, 7, 11	2,6,7,12	0	0	7	8
OCM11	100.00%	100%	11	11	3	1	4	5
OCM12	92.19%	100%	2, 7, 11	12	0	5	5	2
OCM13	68.06%	100%	3, 4, 7	13	0	5	9	2
OCM14	64.22%	86.50%	3, 4, 7	4,6,13	0	0	11	10
OCM15	39.68%	63.20%	3, 4, 7	4,13	0	0	12	12

Correlation Coefficient Calculation between CRS and VRS models of OC Mines

It is worthwhile to interpret the correlation [28] between the various rankings given by CRS & VRS to know the degree of association between various methods. The correlation is calculated using the Spearman's rank correlation coefficient (r_s) as follows:

$$r_s = 1 - \frac{6\sum_{i=1}^{n}(X_i - Y_i)^2}{n(n^2-1)}$$

Where $\mathbf{X_i}$ is the rank of the observation of variable **X** i.e. in CRS model; $\mathbf{Y_i}$ is the rank of the observation of variable **Y** i.e. in VRS model**;** n is the number of pairs of observations.

Correlation Coefficient between Input-oriented CRS and VRS models

X is the Ranks assigned by Input-oriented CRS model.

From table2 in column 8 ranks are $X1_{,}$ $X_{2,}$ ------------ X_{15} = 12, 3, 3, 2,10,4, 1, 8, 6, 7, 4,5,9,11,12.

Y is the Ranks assigned by Input-oriented VRS model.

From table 2 in column 9 ranks are $Y_{1,}$ $Y_{2,}$ ------------ Y_{15} = 6, 3, 5, 1, 7, 2, 4, 11, 9, 8, 5, 2, 2, 10, 12

6 {$(12\text{-}6)^2$ + $(3\text{-}3)^2$ +--
------ + $(12\text{-}12)^2$}
Correlation Coefficient ( ) =1 - ----------------------------------
----
15(15^2- 1)
Correlation Coefficient = 0.7465

Correlation Coefficient between output-oriented CRS and VRS models

X is the Ranks assigned by Output-oriented CRS model.

From table2 in column 8 ranks are $X_{1,}$ $X_{2,}$ ------,X_{15} = 13, 3, 3, 2, 11, 4, 6, 9 ,7, 8, 5, 6, 10, 12, 14

Y is the Ranks assigned by Output-oriented VRS model.

International Journal of Ethics in Engineering & Management Education

Website: www.ijeee.in (ISSN: 2348-4748, Volume 2, Issue 1, January 2015)

From table2 in column 9 ranks are $Y_{1,}\ Y_{2,}$------------- $,Y_{15}$ = 5, 3, 4, 2, 6, 2, 3, 8, 9, 7, 4, 1, 1, 10, 11

$$\text{Correlation Coefficient } (r_s) = 1 - \frac{6\{(13-5)^2 + (3-3)^2 + \text{------} + (14-11)^2\}}{15(15^2-1)}$$

Correlation Coefficient r_s = 0.5929

From above equations high degree of correlation (r_s = 0.7465) between the ranks assigned by input-oriented CRS and input-oriented VRS models has been observed. Similarly, the correlation between rankings of output-oriented CRS and output-oriented VRS models is 0.5929, which indicates a very weak relationship between the rankings of the two models.

CONCLUSIONS

In CRS model 6 units are (OCM2, OCM3, OCM4, OCM6, OCM7 and OCM11) become benchmarking units due to efficiency score approaches to 100% where as in VRS model 8 units are (OCM2, OCM3, OCM4, OCM6, OCM7, OCM11, OCM12 and OCM13) referring units. Two mines OCM12 and OCM13 are inefficient mines in CRS model but those become efficient mines in VRS model.

From the columns two and three in above table the efficiency scores of all the mines according to VRS model greater than the CRS model. For example OCM1 produced 54.96% in CRS model whereas 96.50% in VRS model. There is a lot of improvement of mines taken place to adopt the VRS model for analysis. OCM15 shows very poor performance based on CRS assumption but efficiency increased drastically in VRS model.

Benchmarking units or peer groups also different in both the cases except in case of efficient mines. In OCM9 peer group is 2, 6, 7 in CRS model and 2, 6,7,12 in VRS model, that means 2, 6, 7 three mines are equal in both the cases but not all.

Peer count also different in both the Models except few cases, for example Peer count of OCM4 is 4 in both CRS and VRS model. OCM12 and OCM13 peer count 0 in case of CRS model whereas 5 in case of VRS model due to conversion of in-efficient mines to efficient mines.

Ranking assigned by DEA also different in both the cases except in OCM2 and OCM15 allotted same ranks 3 and 12 respectively in both the cases.OCM7 given 1 rank in CRS but 4 in VRS model. Similarly OCM4 given 1 rank in VRS but 2 in VRS model.

The overall VRS model shows more performance of OC Mines than CRS model. The mean efficiency score for all DMUs shows 0.8178 (81.78%) in CRS model and mean efficiency score for all DMUs shows 0.9392 (93.92%) in VRS model. These scores indicate plenty of scope exists for improvement in OC Mines.

REFERENCES

[1] Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. European Journal of Operational Research, 2, 429-444.

[2] Srinivas Talluri**,** Data Envelopment Analysis: Models and Extensions, 2000, the journal of Production/Operations management.

[3] Banker, R.D., Charnes, A. and Cooper, W.W. (1984), 'Some models for estimating technical and scale inefficiencies in data envelopment analysis', Management Science, Vol. 30, No. 9,pp.1078-1092.

[4] M. Yasin, "The theory and practice of benchmarking: then and now,"Benchmarking: An International Journal, vol. 9, iss. 3, pp. 217-243, 2002.

[5] K. Tone, "A slacks-based measure of efficiency in data envelopment analysis," European Journal of Operational Research, vol. 130, iss. 3,pp. 498-509, 2001.

[6] Andersen P and Petersen NC (1993). A procedure for ranking efficient units in Data Envelopment Analysis. Mngt Sci 39: 1261–1264.